

This question paper contains 4 printed pages]

June 2025

Roll No.

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S. No. of Question Paper : 5783

Unique Paper Code : 2352011201

Name of the Paper : Linear Algebra

Name of the Course : B.Sc. (H) Mathematics

Semester : II / DSC

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt all questions by selecting any two parts from each question.

All questions carry equal marks.

Use of calculator is not allowed.

1. (a) Prove that if x , y and z are mutually orthogonal vectors in $\mathbf{R}^n$, then

$$\|x + y + z\|^2 = \|x\|^2 + \|y\|^2 + \|z\|^2.$$

Also, verify the same for the vectors in $\mathbf{R}^4$, where $x = (1, 0, -1, 0)$, $y = (1, \sqrt{2}, 1, 1)$ and $z = (1, -\sqrt{2}, 1, 0)$.

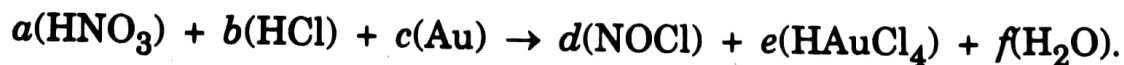
- (b) Solve the following system of linear equations using Gaussian Elimination Method :

$$3x_1 - 2x_2 + 4x_3 = -54$$

$$-x_1 + x_2 - 2x_3 = 20$$

$$5x_1 - 4x_2 + 8x_3 = -83.$$

- (c) Using the Gauss-Jordan method, find the minimal integer values for the variables that will balance the following chemical equation :



P.T.O.

2. (a) Define a linear combination of the vectors in $\mathbf{R}^n$. Express the vectors $[1, 11, -4, 11]$ as a linear combination of the vectors $x = [2, -4, 1, -3]$, $y = [7, -1, -1, 2]$ and $z = [3, 7, -3, 8]$.
- (b) Consider the matrix :

$$A = \begin{bmatrix} 5 & -8 & -12 \\ -2 & 3 & 4 \\ 4 & -6 & -9 \end{bmatrix}$$

- (i) State the Cayley-Hamilton Theorem.
- (ii) Find the eigenvalues and eigenvectors of A.
- (iii) Check whether the matrix A is diagonalizable or not ?
- (c) Let

$$A = \begin{bmatrix} 2 & 3 & 4 \\ 0 & 1 & 1 \\ -2 & 1 & 5 \\ 3 & 0 & 1 \end{bmatrix} \text{ and } B = \begin{bmatrix} 2 & 1 & -5 \\ 2 & 3 & 0 \\ 4 & 1 & 1 \end{bmatrix}$$

Compute $R(AB)$ and $(R(A))B$ to verify that they are equal, i.e., $R(AB) = (R(A))B$, for the operations :

- (i) $R : \langle 3 \rangle \leftarrow -3 \langle 2 \rangle + \langle 3 \rangle$,
- (ii) $R : \langle 2 \rangle \leftrightarrow \langle 3 \rangle$.

3. (a) Let $\mathbf{R}^2 = \{(a_1, a_2) : a_1, a_2 \in \mathbf{R}\}$. Prove that $\mathbf{R}^2$ is a vector space over $\mathbf{R}$ with addition and scalar multiplication defined as;

$$(a_1, a_2) + (b_1, b_2) = (a_1 + b_1, a_2 + b_2)$$

$$\alpha(a_1, a_2) = (\alpha a_1, \alpha a_2) \text{ where } (a_1, a_2), (b_1, b_2) \in \mathbf{R}^2, \alpha \in \mathbf{R}.$$

- (b) Define basis of a vector space. Prove that the following subset S of $\mathbf{R}^3$ is a spanning set of $\mathbf{R}^3$, where $S = \{(2, 2, 3), (-1, -2, 1), (0, 1, 0)\}$.
- (c) Do the polynomials $x^3 - 2x^2 + 1$, $4x^3 - x + 3$ and $3x - 2$ generates $P_3(\mathbf{R})$?
4. (a) If $\{v_1, v_2, \dots, v_n\}$ is a basis of a vector space $V(F)$, then prove that every element of V can be uniquely expressed as a linear combination of $v_1, v_2, \dots, v_n$.
- (b) Define linearly dependent and independent set in $\mathbf{R}^3$. Show that set of vectors $\{(2, -2, 3), (0, -4, 1), (3, 1, -4)\}$ in $\mathbf{R}^3$ is linearly dependent.
- (c) Let W be a subspace of a finite-dimensional vector space V . Prove that W is finite-dimensional and $\dim(W) \leq \dim(V)$. Moreover, if $\dim(W) = \dim(V)$, then $V = W$.
5. (a) Let $T : \mathbf{R}^2 \rightarrow \mathbf{R}^2$ is a function. For each of the following parts, state why T is not a linear transformation.
- (i) $T(a, b) = (1, b)$
 - (ii) $T(a, b) = (a, b^2)$
 - (iii) $T(a, b) = (\sin a, 0)$
 - (iv) $T(a, b) = (|a|, b)$
 - (v) $T(a, b) = (1 + a, b)$.
- (b) Let V, W and Z be finite-dimensional vector spaces with ordered basis α, β and γ , respectively. Let $T : V \rightarrow W$ and $U : W \rightarrow Z$ be linear transformations. Then $[UT]_{\alpha}^{\beta} = [U]_{\alpha}^{\beta} [T]_{\alpha}^{\beta}$.
- (c) Prove that two same dimensional finite vector spaces over the same field are isomorphic.

6. (a) Let V and W be vector spaces over same field F , and suppose that $\{v_1, v_2, \dots, v_n\}$ is a basis for V . For $w_1, w_2, \dots, w_n$ in W , there exists exactly one linear transformation $T : V \rightarrow W$ such that $T(v_i) = w_i$ for $i = 1, 2, \dots, n$.

- (b) Let $L : M_2 \times_2(\mathbf{R}) \rightarrow \mathbf{R}^3(\mathbf{R})$ be a linear transformation given by

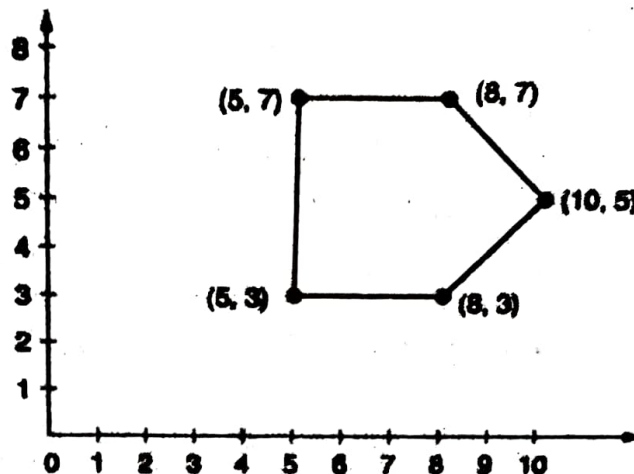
$$L\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = [a - c + 2d, 2a + b - d, -2c + d] \text{ with}$$

$$B = \left(\begin{bmatrix} 2 & 5 \\ 2 & -1 \end{bmatrix}, \begin{bmatrix} -2 & 2 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -3 & 4 \\ 1 & 2 \end{bmatrix}, \begin{bmatrix} -1 & -3 \\ 0 & 1 \end{bmatrix}; \right)$$

$$C = ([7, 0, -3], [2, -1, -2], [2, 0, 1]).$$

Check whether B and C are bases for $M_2 \times_2(\mathbf{R})$ and $\mathbf{R}^3(\mathbf{R})$, respectively, if yes, then find matrix A for L with respect to the given bases B and C .

- (c) For the graphic in given figure, use ordinary coordinates to find the new vertices after performing each indicated operations :



- (i) Translation along the vector $[4, -2]$
- (ii) Rotation about the origin through $\theta = 30^\circ$
- (iii) Reflection about the line $y = 3x$.

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(17/6/25 ✓

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S. No. of Question Paper : 5803

Unique Paper Code : 2352011202

Name of the Paper : Calculus-DSC 5

Name of the Course : Bachelor of Science (Honours Course)
Mathematics NEP UGCF-2022

Semester : II

Duration : 3 Hours

Maximum Marks : 90

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Attempt *all* questions by selecting *three* parts from each question.

All questions carry equal marks.

Use of calculator is not allowed.

1. (a) Using $\varepsilon - \delta$ definition of limit, show that :

$$\lim_{x \rightarrow c} \frac{1}{x} = \frac{1}{c},$$

if $c > 0$.

- (b) Show that if $f : (a, \infty) \rightarrow \mathbf{R}$ ($a > 0$) is such that

$$\lim_{x \rightarrow \infty} xf(x) = L,$$

where $L \in \mathbf{R}$, then

$$\lim_{x \rightarrow \infty} f(x) = 0.$$

P.T.O.

- (c) Let $A \subset \mathbf{R}$ and $f : A \rightarrow \mathbf{R}$ has a limit at $c \in \mathbf{R}$, where c is a cluster point of A . Then prove that f is bounded on some neighbourhood of c .
- (d) Give example of functions f and g such that f and g do not have limits at a point c , but both $f + g$ and fg have limits at c .
2. (a) If f is continuous on a closed interval $[a, b]$, then prove that f is uniformly continuous on $[a, b]$.
- (b) Show that the function $f(x) = x^2$ is uniformly continuous on $[-4, 4]$.
- (c) Show that :

$$\lim_{x \rightarrow \infty} \frac{x+2}{\sqrt{x}} \quad (x > 0)$$

does not exist in $\mathbf{R}$.

- (d) Use $\varepsilon - \delta$ definition of limit to show that :

$$\lim_{x \rightarrow 6} \frac{x^2 - 3x}{x + 3} = 2.$$

3. (a) Let

$$f(x) = \begin{cases} x^2, & x \geq 0, \\ 0, & x < 0. \end{cases}$$

Calculate f' on $\mathbf{R}$. Is f' continuous as well as differentiable on $\mathbf{R}$? Justify your answer.

- (b) Let $f(x) = \sin |x|$, where $x \in \mathbf{R}$. Determine the set of points where f is not differentiable. Justify your answer.

(c) If $f : \mathbf{R} \rightarrow \mathbf{R}$ is differentiable at $a \in \mathbf{R}$, then show that :

$$(i) \quad \lim_{h \rightarrow 0} \frac{f(a+h) - f(a)}{h} = f'(a),$$

$$(ii) \quad \lim_{h \rightarrow 0} \frac{f(a+h) - f(a-h)}{2h} = f'(a).$$

(d) Suppose f is defined on an open interval (a, b) . Let $x_0 \in (a, b)$. If f assumes its maximum at x_0 and if f is differentiable at x_0 , then show that $f'(x_0) = 0$.

4. (a) State and prove Rolle's theorem.

(b) Let f be defined on $\mathbf{R}$ and suppose that $|f(x) - f(y)| \leq |x - y|^3$ for all $x, y \in \mathbf{R}$. Prove that f is a constant function on $\mathbf{R}$.

(c) Show that $\sin x \leq x$ for all $x \geq 0$.

(d) State Intermediate value theorem for derivatives. Suppose f is differentiable on $\mathbf{R}$ and $f(0) = 1$, $f(1) = 2$ and $f(2) = 2$.

(i) Show that $f'(x) = \frac{1}{2}$ for some $x \in (0, 2)$,

(ii) Show that $f'(x) = \frac{1}{3}$ for some $x \in (0, 2)$.

5. (a) If

$$y = \log \left(x + \sqrt{1 + x^2} \right),$$

show that

$$(1 + x^2)y_{n+2} + (2n + 1)xy_{n+1} + n^2y_n = 0.$$

- (b) Trace the curve :

$$4x^2 = y^2(x^2 - 4).$$

- (c) Determine the intervals of concavity and points of inflexion (if any) of the curve :

$$y = 2x^4 - 3x^2 + 2x + 1.$$

- (d) State Taylor's Theorem. Find the Taylor's series expansion of $\sin 2x$ about $x = 0$.

6. (a) Trace the curve :

$$r = 2 \sin 2\theta.$$

- (b) Determine the position and nature of the double points of the curve :

$$x^3 + 2x^2 + 2xy - y^2 + 5x - 2y = 0.$$

- (c) Find the horizontal asymptotes of the graph of the function given by

$$y = x^5 \left[\sin \frac{1}{x} - \frac{1}{x} + \frac{1}{6x^3} \right].$$

- (d) Find the equations of tangents at origin to the curve :

$$x^4 + y^4 = a^2(x^2 - y^2).$$

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Name of the Course : Bachelor of Science (Honours Course)

Mathematics NEP UGCF-2022

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Duration : 3 Hours

Maximum Marks : 90

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S. No. of Question Paper : 5823

Unique Paper Code : 2352011203

Name of the Paper : Ordinary Differential Equations

Name of the Course : B.Sc. (Hons.) Mathematics

Semester : II/DSC

Duration : 3 Hours

Maximum Marks : 90

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the questions are compulsory.

Attempt any two parts from each question.

Each part carries 7.5 marks.

Use of non-programmable scientific calculator is allowed.

1. (a) Solve the initial value problem :

$$(2y \sin x \cos x + y^2 \sin x) dx + (\sin^2 x - 2y \cos x) dy = 0, y(0) = 3.$$

(b) Solve :

$$(x^2 - 3y^2) dx + 2xy dy = 0.$$

(c) Solve :

$$x^2 y'' = 2xy' + (y')^2$$

by reduction of order.

2. (a) If the population of a country double in 50 years, in how many years will it triple under the assumption that the rate of increase is proportional to the number of inhabitants ?

P.T.O.

(2)

- (b) A metal bar at a temperature of 100°F is placed in a room at a constant temperature of 0°F . If after 20 minutes the temperature of the bar is half, find an expression for the temperature of the bar at any time.
- (c) Assume that the rate at which radioactive nuclei decay is proportional to the number of nuclei in the sample. In a certain sample 10% of the original number of radioactive nuclei have undergone disintegration in a period of 200 years. What percentage of the original radioactive nuclei will remain after 1000 years ?
3. (a) Show that e^{2x} and e^{3x} are linearly independent solutions of the differential equation :

$$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0.$$

Find the particular solution satisfying the initial condition,

$$y(0) = 0, y'(0) = 0.$$

- (b) Solve the differential equation using the method of variation of parameters

$$\frac{d^2y}{dx^2} + 4y = \tan 2x.$$

- (c) Find the general solution of the differential equation using the method of undetermined coefficients :

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2e^x.$$

4. (a) Use operator method to find the general solution of the following linear system :

$$\frac{dx}{dt} + 2y + x = e^t$$

$$\frac{dy}{dt} + 2x + y = 3e^t.$$

- (b) Solve the initial value problem, assuming $x > 0$

$$x^3 \frac{d^3 y}{dx^3} - x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} - 2y = x^3.$$

- (c) A mass of 3 kg is attached to the end of a spring that is stretched 20 cm by a force of 15 N. It is set in motion with initial position $x_0 = 0$ m and initial velocity $v_0 = -10$ m/s. Find the amplitude, period and frequency of the resulting motion.

5. (a) Define carrying capacity. Derive the logistic equation :

$$\frac{dX}{dt} = rX \left(1 - \frac{X}{K} \right)$$

where r and K are reproduction rate and carrying capacity respectively.

- (b) Suppose a soft drink manufacturing company has a 1000 litre tank containing sugar water solution. Initially s_0 kg of sugar is dissolved. Sugar solution flows into the tank at the rate of 10 litre/min with concentration $cin(t)$ kg/l. Assume that the solution in the tank is thoroughly mixed and the sugar solution flows out at the same rate at which it flows in i.e. the volume of sugar water mixture in the tank remains constant. Find a differential equation for the amount of sugar in the tank at any time t . Also solve it.

- (c) Suppose a population can be modelled using the differential equation :

$$\frac{dx}{dt} = 0.2x - 0.001x^2$$

with the initial population size of $x_0 = 100$ at a time step of 1. Find the predicted population after two months.

6. (a) The given differential equation :

$$\frac{dN}{dt} = aN \left(1 - \frac{N}{N_T} \right)$$

represents the model for spread of technology where $N(t)$ is the number of ranchers who adopted an improved pasture technology in Uruguay and N_T is the total population of ranchers. It is assumed that rate of adoption is proportional to both the number who have adopted the technology and the fraction of population of ranchers who have not adopted the technology.

- (i) Which term corresponds to the fraction of the population who have not yet adopted the improved pasture technology ?
- (ii) According to the Banks (1994), $N_T = 17015$, $a = 0.490$ and $N_0 = 141$. Determine how long it takes for the improved pasture technology to spread to 80% of the population.
- (b) Derive the epidemic model of influenza

$$\frac{dS}{dt} = -\beta SI, \quad \frac{dI}{dt} = \beta SI - \gamma I, \quad \frac{dR}{dt} = \gamma I$$

where β and γ are positive constants.

- (c) Define equilibrium points. Find all equilibrium points of Predator-Prey model

$$\frac{dX}{dt} = \beta_1 X \left(1 - \frac{X}{K} \right) - c_1 XY, \quad \frac{dY}{dt} = c_2 XY - \alpha_2 Y$$

where the positive constants c_1 , c_2 are interaction parameter, α_2 is predator per capita death rate and β_1 is prey per capita birth rate. Discuss the direction of trajectories of Predator-Prey model.

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Name of the Course : B.Sc. (Hons.) Mathematics

Semester : II/DSC

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UNIVERSITY OF DELHI

CORRIGENDUM

Sl. No. of Q.P.	:	5823
Unique Paper Code	:	2352011203
Name of the Paper	:	Ordinary Differential Equations
Course	:	B.Sc. (Hons.) Mathematics
Exam	:	18/06/2025 Morning
SEMESTER/YEAR	:	II/DSC
DURATION	:	3 Hour
Maximum Marks	:	90

Announcement for candidates

Q. 4 be read as

(a) Use operator method to find the general solution of the following linear system

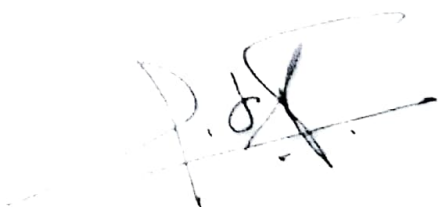
$$\frac{dx}{dt} + 2y + x = e^t$$

$$\frac{dy}{dt} + 2x + y = 3e^t$$

(b) Solve the initial value problem, assuming $x \geq 0$

$$x^4 \frac{d^4 y}{dx^4} - x^2 \frac{d^2 y}{dx^2} + 2x \frac{dy}{dx} + 2y = x^2$$

(c) A mass of 3 kg is attached to the end of a spring that is stretched 20 cm by a force of 15 N . It is set in motion with initial position $x_0 = 0 \text{ m}$ and initial velocity $v_0 = -10 \text{ m/s}$. Find the amplitude, period and frequency of the resulting motion.



Joint Registrar (Secrecy)